Statistical Optical Link Analysis

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Outline of Talk

Introduction

Statistical Link Analysis

- **Statistical RF Link Analysis**
- **Extension to Statistical Optical Link Analysis**
  - Sources of Signal and noise power fluctuations
  - Optical link equation and link margin

Simulations and modeling

- **Signal and noise power modeling**
- **Statistically adjusted Coded performance of SCPPM and margin**

Conclusion and Future Work

Introduction

- Statistical link analysis is a useful tool to leverage knowledge about link parameter uncertainties to optimize achievable data rates
  - Traditional Statistical RF Link Analysis
    - Additive nature of RF link equation (in dB domain)
    - Gaussian approximation used for the received Signal to Noise Ratio (SNR) for quiescent links, e.g. X-band, S-band, UHF, etc.
    - Link margin policies are based on the standard deviation of the SNR.
  - Statistical Optical Link Analysis
    - No closed form representation that maps the received signal power and noise power to achievable data rates
    - Optical link uncertainties often have non-Gaussian, dominant link parameters, e.g., pointing or turbulence induced fading
- The fundamental differences between an optical link equation and RF link equation does not lend us to develop a straightforward extension
  - We simulated and obtained coded performance curves for a fairly wide range of operational signal and noise levels
  - We adopted numerical methods to incorporate signal and noise power distribution models
- Simulation results reveal some interesting facts about effects of signal and noise fluctuation for several different operational scenarios.

Statistical Link Analysis: RF links

- RF link equation
  - The capacity of power constrained RF Gaussian Channel (for quiescent links) is a linear function of received Signal to Noise Ratio

$$C_{RF} \approx \frac{1}{\ln(2)} \frac{P_r}{kT} \text{ (b/s)}$$

- RF link analysis tabulates the signal and noise power to estimate the link SNR, which can be represented as an additive equation in the dB domain

$$\frac{P_r}{kT} = EIRP + G - kT - L_{range} - L_o$$

EIRP : effective isotropic radiator power of the transmitter.

G: includes all the gain and loss terms on the receiver side

L_{range}: space loss

L_o: loss incurred from transmission media, e.g., atmospheric loss

- Traditional Statistical RF Link Analysis (for quiescent links)
  - Gaussian approximation can be used for the received Signal to Noise Ratio (SNR)
    - Mean and variance equal to the sum of the means and variances of the link parameters
      - Central limit theorem, and Lyapunov's condition
    - Link can be designed based on the statistical confidence levels measured in terms of the standard deviation of received SNR.

Statistical Link Analysis: Optical links

- The capacity of the Poisson PPM channel

$$C_{OPT} = \frac{1}{\mathrm{M}T_S}\left(D(p_1||p_0) - \mathrm{D}(p_y||p_{y|0})\right) \text{ (b/s)}$$

$D(f||g)$: relative entropy

$p_{0,}\, p_{1,}$: probability mass functions of a noise and signal slot

$p_{y,}\, p_{y|0,}$: probability mass functions of a random PPM symbol and a noise vector

- Except for the noiseless case, this equation requires approximation of a multi-dimensional infinite sum or a Monte-Carlo simulation
- An approximate optical PPM link equation exists which combines a number of bounds on optical capacity

$$C_{OPT} \approx \frac{1}{\varepsilon \ln(2)}\left(\frac{{P_r}^2}{P_r\frac{1}{\ln(M)} + P_n\frac{2}{M-1} + {P_r}^2\frac{MT_S}{\ln(M)\varepsilon}}\right)$$

$P_{r,}\, P_{n,}$: detected signal and noise power

- Statistical Optical Link Analysis
  - Straightforward extension of statistical analysis does not apply unless we define either signal-dominant or noise dominant regime.
  - During a deep space optical link pass, it may sweep over both regimes.
  - We need to perform separate modeling of signal and noise distributions

Sources of uncertainties: signal power budget

- Received signal power in a free space optical link (in dB domain)

$$P_r = P_t + G_t + G_r - L_s - L_{atm} - L_{pt} - L_t - L_r$$

P_t : transmitted power
G_t : transmitter gain
G_r : receiver gain
L_s : range loss
L_{atm} : atmospheric loss
L_{pt} : pointing loss
L_t : transmitter related loss
L_r : receiver related loss

- Space and Ground Terminals
  - Transmitter related loss
    - Optical coupling loss, propagation loss through the optical system, etc.
  - Receiver related loss
    - Filter transmission loss, polarization loss, and coupling loss
  - The distribution models of terminal related losses are system specific
    - They can be measured and characterized once the transmitter and receiver design is determined.

Sources of uncertainties: signal power (cont'd.)

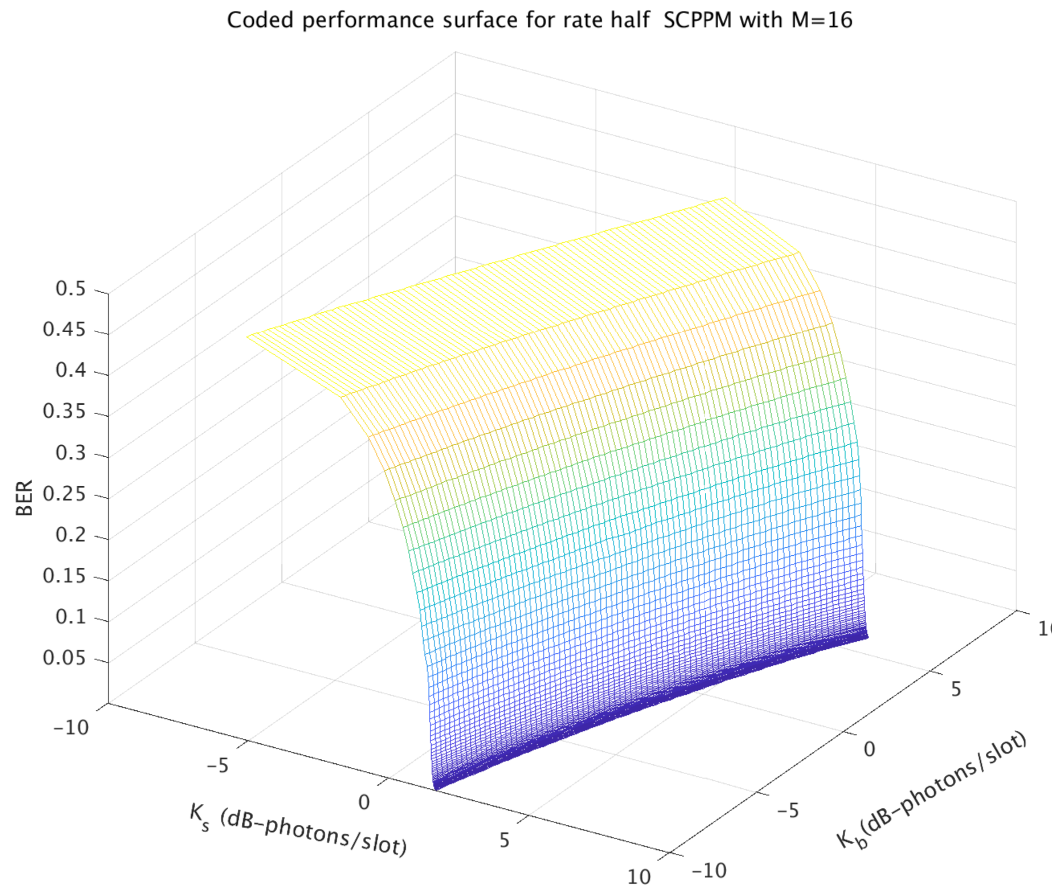
- Pointing Error and Jitter
  - Pointing loss
    - Pointing error results in a static loss
    - Pointing jitter causes uncertainties in the pointing loss term, leading to pointing-induced fading in received signal power
- Atmospheric loss
  - Signal attenuation caused by absorption and scattering
  - The MODerate resolution atmospheric TRANsmission (MODTRAN) simulation tool can be used to predict the distribution model of the atmospheric loss under a range of weather conditions.
- Turbulence-induced fading
  - Clean air turbulence causes random fluctuations in received signal power
  - Beam wandering effects also lead to variations in encircled signal power
    - Adaptive optics and aperture averaging can reduce these effects
  - Analysis models exist for the fading process
    - The turbulence refraction index can be inferred based on weather availability assumption

Sources of uncertainties: noise power

- Noise source
  - Incident background light
  - Detector dark currents
  - Thermal noise
- During day time passes, background noise is dominated by sky radiance
  - NASA's AERONET historically monitors day time sky radiance at various locations
  - The empirical data set can be used to produce the noise distribution model for any given geometry, location, and atmospheric conditions.
- Night time passes
  - Signal dominant regime

Deterministic Optical Link Analysis: Link Margin

- Link Margin in standard optical link analysis
  - Coded performance curves
    - Chosen error correction code, e.g., SCPPM with chosen code rate and PPM order
    - Range of average noise power values
  - Required signal power is derived from the coded performance curves
    - Operational data rate (Code rate and PPM order)
    - Tolerable code word error rate
  - Received signal power
    - Signal power budget
  - Link margin to decide link closure, e.g., 3-dB margin is typically used to accommodate uncertainties in the link.



- Produced using CCSDS SCPPM prototype software
- Operational data rate is determined by the code rate, PPM order, and slot width
- The region above coded performance surface is the achievable region

Modeling and simulation: Statistically adjusted performance

- Statistically adjusted coded performance
  - Given the deterministic coded performance surface $h(x, y)$, x and y are the average signal and noise photon flux rates (photons/slot)
  - Statistically adjusted coded performance can be obtained by convolving the deterministic performance with the signal and noise distributions
    - Assuming signal and noise flux rates are independent of each other, with marginal distributions of $f(x)$ and $g(y)$

$$\bar{h}(x,y) = \iint h(x,y)\, f(x) g(y) dx\, dy$$

- Signal flux rate distribution
  - System-related effects: Space and ground terminals, pointing-induced fading, etc.
  - Weather-related effects: Atmospheric transmittance, turbulence-induced fading
- Background noise flux rate distributions
  - Sky Radiance
- Weather availability models can be incorporated into the modeling using weighted sum of conditional distributions to model signal and noise

Modeling and Simulation: Example (1)

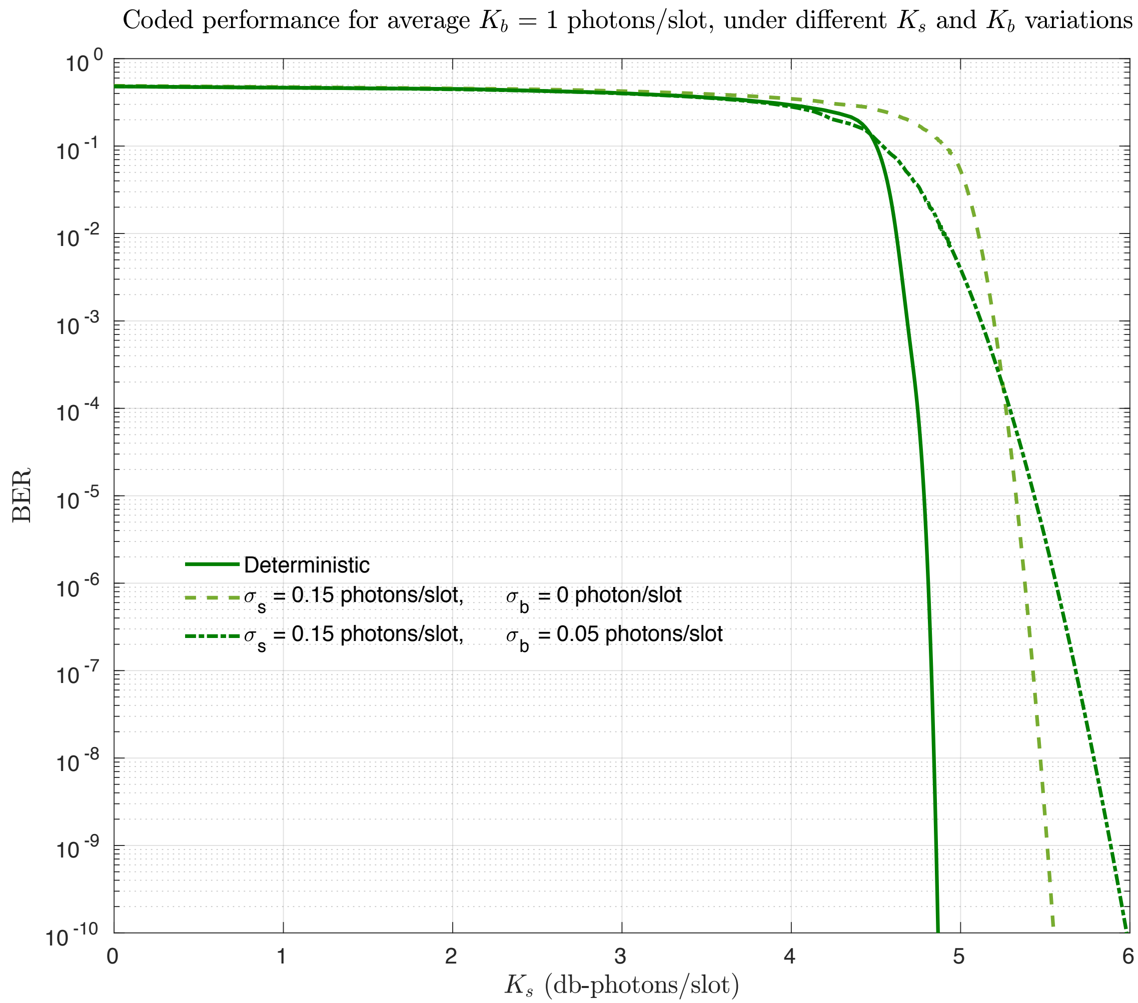
- Received signal is modulated by log-normal distribution : $P_r = K_s v(t)$ where

$$f_V(v) = \frac{1}{\sqrt{2\pi\sigma_I^2}} \frac{1}{v} \exp\left(\frac{-(\ln v + \sigma_I^2/2)^2}{2\sigma_I^2}\right)$$

- Average noise flux rate is
  K_b = 1 photon/slot (nominal)
  - Deterministic
  - Gaussian distributed with
    σ_b = 0.05 photons/slot

In this nominal background noise case

- When only signal fluctuation is present
  - Fading loss: coded performance shifted to the right
- When both signal and noise fluctuations are present
  - Gap widens towards the high SNR region

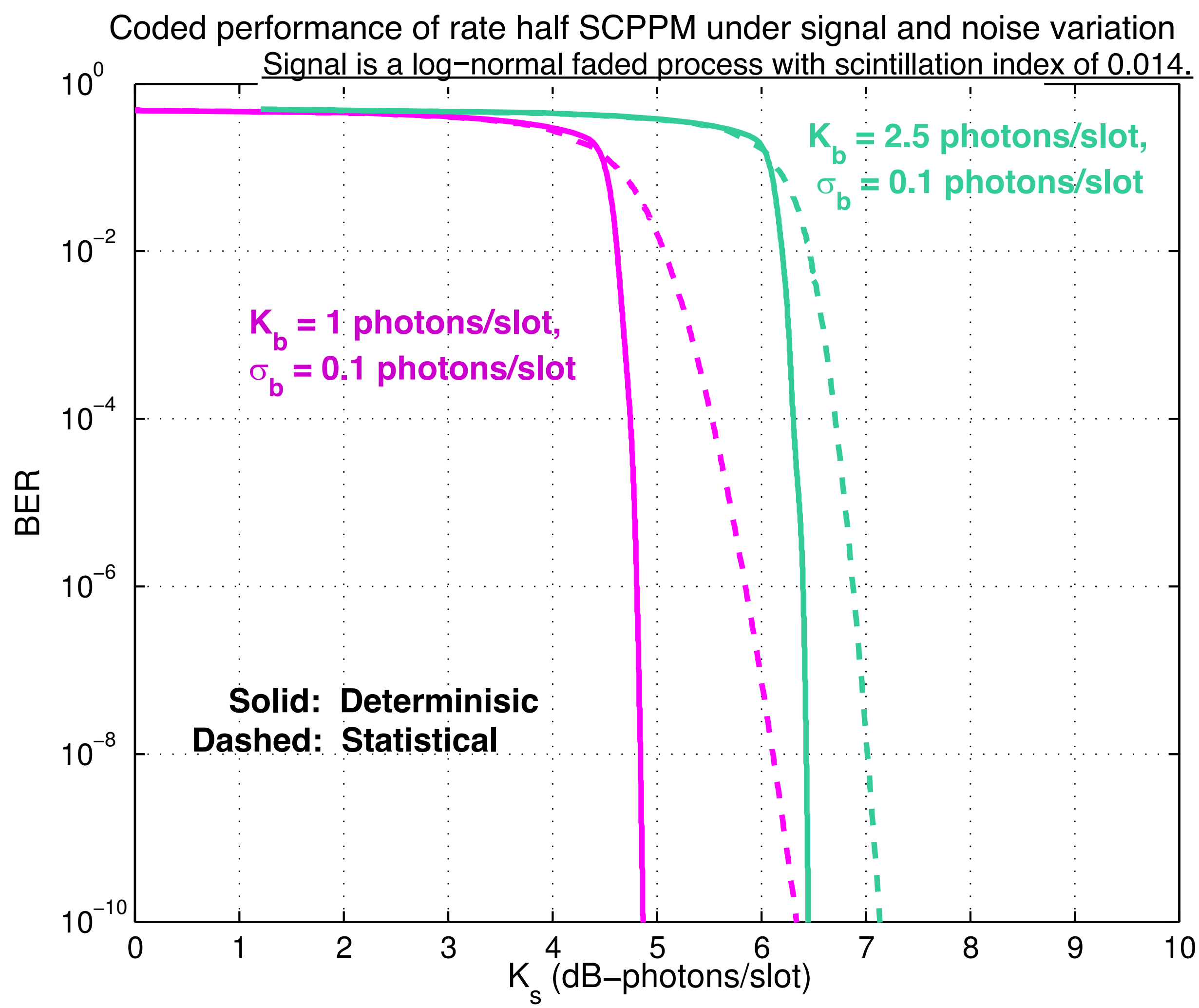


Modeling and Simulation: Example (2)

Effects of turbulence-induced fading at different background noise levels

- Signal is a log-normal faded process with scintillation index of 0.014.
- Noise background is Gaussian distributed with standard deviation of 0.1 photons/slot
  - Nominal noise background
    - K_b = 1 photon/slot
  - Adverse noise background (small SEP angle, for example)
    - K_b = 2.5 photon/slot

- Effects of signal power fluctuation
  - More profound on nominal case where the tail of the noise distribution causes a transition to noise dominant regime

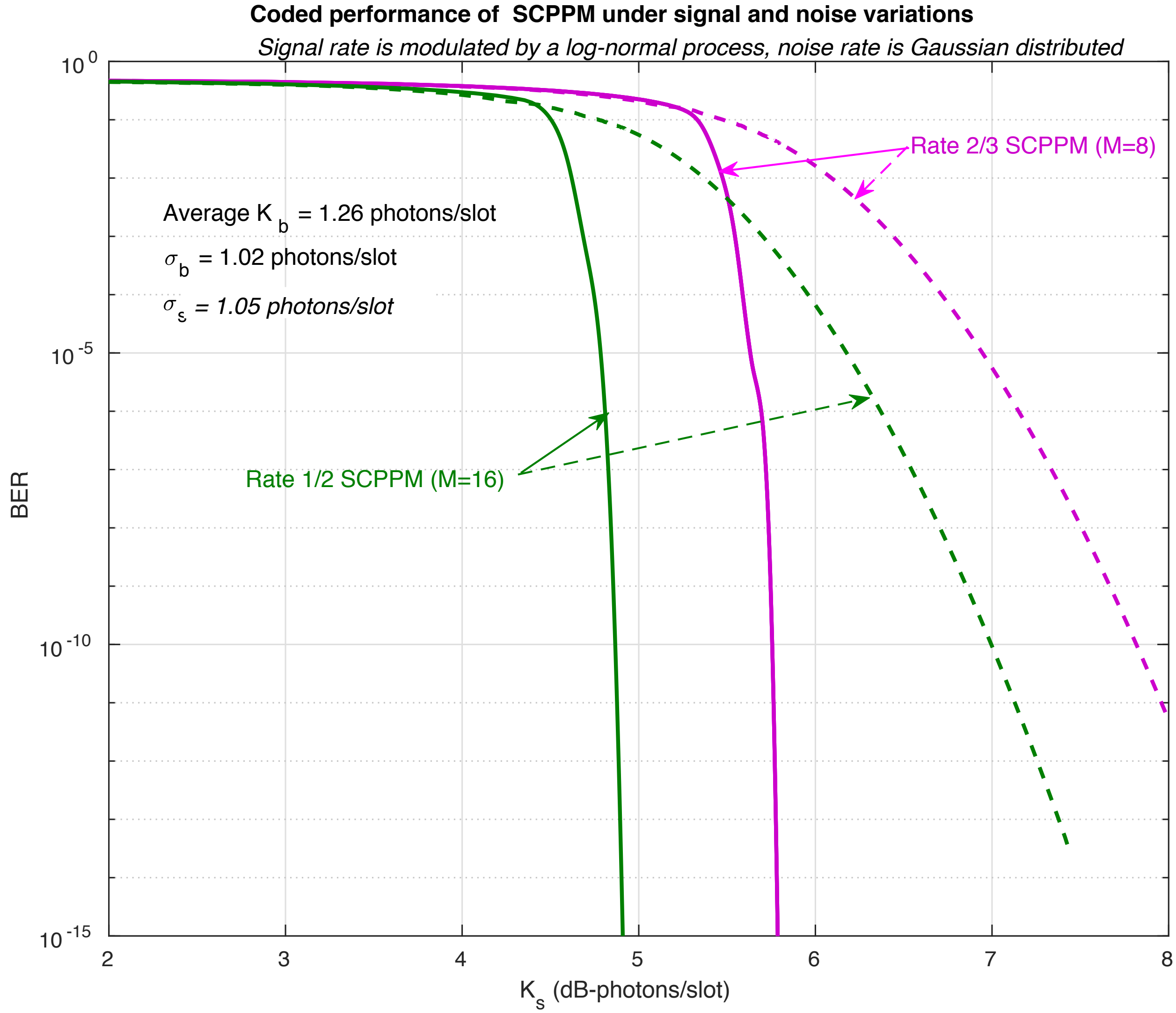


Modeling and Simulation: Example (3)

Statistically adjusted coded performance curves at different data rates

- Nominal noise background
  - K_b = 1.26 photon/slot
  - σ_b = 1.02 photon/slot
- Signal modulated by a log-normal distribution with
  σ_s = 1.05 photon/slot

- SCPPM
  - r = ½, M = 16
  - r = 2/3, M = 8
- Effects of signal and noise fluctuations on coded performances are similar under these two configurations



Conclusion and future work

- Extension of statistical analysis framework to optical communication links
  - Performed analysis on the relationship between
    - Bit error rate requirements
    - Statistical characterization of the signal and noise power
    - Coded performance of SCPPM
  - Link analysis results
    - Intensity modulated, direct detected photon-counting channel utilizing PPM
    - Preliminary uncertainty quantifications of the signal and noise power
    - Effects of channel turbulence and background noise fluctuation can vary significantly depending on the regime of the operational points
      - Geometry, weather, operational data rate, etc.
- We plan to seek and obtain historical weather data and incorporate the CDF statistics into link analysis